FEATURES

Options in Real Estate Valuation

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ABSTRACT

Real options valuation theory is well developed and often used in financial markets, but infrequently applied in real estate appraisal. Real estate presents several opportunities to employ real option valuation theories, including real estate purchase options, development and acquisition feasibility, bankruptcy, abandonment, and litigation. Today, options theory could be employed whenever an appraiser advises a client about a future real estate decision over substantially different and incompatible risky choices. This article explains real options analysis and valuation theories as they apply to real estate, including financial option pricing models, the Black-Scholes-Merton model, real options analysis, binomial options models, and Monte Carlo simulation.

Real estate purchase options[1] and real option[2] issues arise regularly in the real estate industry and within the context of real estate valuation. Real option valuation methods and financial option pricing models are superb valuation methods. The body of knowledge is advanced in other financial industries and offers underused practical methods for practicing real estate appraisers. In some important cases, the methods are superior to conventional appraisal techniques, because they allow decision makers to more accurately estimate value for certain types of assets. Real options valuation and financial option pricing models are superior for higher-risk assets, assets with uncertain valuation inputs, and/or assets that have significant incompatible decision options. These valuation methods explicitly evaluate the flexibility, options, and choices inherent in an asset and account for risk and uncertainty in ways that conventional appraisal methods do not.

This article explains the concepts behind real options and financial option valuation theories, and expands on conventional real estate appraisal theory to encompass real option and financial option valuation theories. The article demonstrates that option valuation theories and methods can be employed on traditionally problematic appraisal problems, such as appraising the value of a real estate purchase option and options surrounding bankruptcy, abandonment, and feasibility and development planning. Examples of options valuation are given for a land subdivision and a real estate purchase option. The article also provides an introductory translative bridge between the large body of knowledge on options valuation in the financial industries and the real estate community, since each uses different terminology and notation.

Real Options

In short, real options are choices that have significant and very different consequences, where the choices will be made in the future, based on future conditions that are uncertain as of today. Real options in the financial and

1. Appraisal Institute, *The Dictionary of Real Estate Appraisal*, 5th ed. (Chicago: Appraisal Institute, 2010), 139, defines *option* as it relates to real estate as "a legal contract, typically purchased for a stated consideration, that permits but does not require the holder of the option (known as the *optionee*) to buy, sell, or lease real property for a stipulated period of time in accordance with specified terms; a unilateral right to exercise a privilege.

2. A *real option* is the right, but not the obligation, to execute a specific business decision within a specified time frame.

valuation sense are options to choose to do either one thing or another at some future time, each choice being substantially different and mutually exclusive of the other. Each choice will have substantially different financial and value-creating consequences where, as of today, it is unclear which choice should be made. The choice may be to buy, sell, rent, abandon, foreclose, condemn, switch, swap, expand, shrink, or not. The choice can dramatically affect valuation inputs and outputs. The choice will be made in the future after the valuation date based on the then-current conditions, which as of the valuation date can only be forecasted.

As the uncertainty concerning the forecasts increases, the impact of the real options on value increases. For example, appraisers know that whether an existing tenant chooses to renew or not can have dramatic effects on the value of a property. As the uncertainty of the tenant's renewal increases, so does the impact on value. Similarly, a real estate option owner will decide to exercise a real estate purchase option at a future option exercise date. If exercised, the value of the real property will be wholly different from the value if the decision is made to not exercise the option. A common type of option found in real estate markets is the land option. Purchasing an option on land allows developers to gain control of a property and lock in a price while exercising their due diligence to determine if the property will be suitable for their development plans.

In finance, an *option* is a contract that gives the buyer of the option the right to buy (a *call*), or gives the seller the right to sell (a *put*), an underlying asset at a specified price (the *exercise price*), on or before the end a specified time period (the *term*). The value of an option is composed of two parts: the intrinsic value of the option and the time value of the option, which is primarily related to the uncertainty of future price movements of the underlying asset. This uncertainty is itself based on a number of factors, including the time (or term) of the option remaining until expiration, the difference between the exercise price and the value of the underlying asset, and the future volatility of the value of the underlying asset.[3]

Financial options have real options. Financial options are a type of investment whose value is derived from (hence the term *derivatives*) the value of an underlying asset and where a real option is created for the investor. The value of a real estate purchase option (which is a financial option) is derived from the value of the underlying real estate (fundamental asset). A real estate purchase option presents the option buyer with a real option, which is the choice to exercise or let expire the right to buy the underlying real estate. The value of that option relates to the underlying value of the real estate, the exercise price, the term of the option, the option price, and the risk and yield. The word *real* in real estate and in real options are not related.[4]

Literature Review

Achour and Brown[5] in their article conclude that option valuation modeling can and should be considered when valuing land options. The authors point out that the majority of land options are used to gain control of the property rather than as a speculative investment like a stock option. They also emphasize the potential difficulty in estimating the proper variance to use in real property option valuation.

In their 1987 article, "On Option-Pricing Models in Real Estate: A Critique," Shilling, Sirmans, and Benjamin explore some of the conceptual problems and empirical issues with applying an options valuation methodology to real estate.[6] Conceptual issues include institutional forces specific to real estate, such as higher transaction costs, longer transaction periods, relatively illiquid markets, and the fact that real estate options are frequently purchased in an effort to gain some control over the property rather than as speculative investments with an intent to arbitrage fluctuations in the price of the underlying asset. Empirical issues include practical difficulties in applying option pricing models to real estate, such as measuring the current value of the underlying asset, measuring the variance applicable to the underlying asset, and collecting relevant data to the problem at hand. The authors conclude that while a variety of real estate decisions can be

3. The volatility of the value of the asset is frequently described in terms of the sigma (σ), which is the standard deviation of the value or returns.
4. The words *option* in *real option* and in *financial option* or *stock option* are related, but there are distinctions that will not be explored here.
5. Dominique Achour and Robert L. Brown, "Appraising Land Options," *The Real Estate Appraiser and Analyst* 50, no. 2 (Summer 1984): 62—66.
6. James D. Shilling, C. F. Sirmans, and John D. Benjamin, "On Option-Pricing Models in Real Estate: A Critique," *AREUEA Journal* 15, no. 1 (1987): 742—752.

modeled using options-pricing techniques, special consideration needs to be given to the conceptual, practical, and empirical issues that arise.

Most real estate decisions do not present material real options. In most cases, lowering the rental rate by one dollar will not dramatically change the value of a property and the owner's actions. Usually the property's rental rate will decline modestly by one dollar and its value will decline proportionally, modestly, and without significant consequence. On some occasions, however, the loss of that one dollar will drive the property into bankruptcy, and then there is a real option to be accounted for in the valuation exercise, because deciding today about what to do about a future going concern is different than dealing with a future bankruptcy.

Colwell and Colwell present another way to think of real options in real estate: as a contribution to the real estate's overall value.[7] Just as the overall value can be broken into the value of land plus building or the value of mortgage plus equity or the value of cash flow plus reversion, the overall value of real estate that includes options value can be broken down into components as shown in the following equation.

$$V_o = V_{f/i} + V_{option}$$

where,

V_o = Overall value of the real estate

$V_{f/i}$ = Fundamental or intrinsic value of the real estate

V_{option} = Value of the option included in the overall value

The notions of fundamental and intrinsic values, while undeveloped in real estate,[8] are well established in financial options theory.[9] The major difference is the accounting for change over time. Financial options account for the fact that fundamentals can change dramatically with time, while traditional real estate valuation theory assumes the fundamentals are knowable and the outcomes do not change once forecasted. This is why real estate valuation has traditionally been described as deterministic. As the Colwell and Colwell article indicates, if the interim highest and best use of a parcel is as a farm, its fundamental or intrinsic value is based on the productivity of the farm use but only if the fundamentals do not change. However, if years from now the highest and best use might change to something that creates a higher value, say a residential subdivision, then a seller today should sell at a value indicated by productivity of the farm use plus the incremental value enhancement from the potential for the change in use. Such a buyer is acquiring the value of the farm productivity and the value of the option on the potential for a change.

When option value actually exists, it is intrinsically reflected in the sales comparison approach (assuming the comparables chosen enjoy similar real options and current and future highest and best uses). The income capitalization approach may not intrinsically reflect the real options. Many appraisers are familiar with the reconciliation issues surrounding properties nearing a change in use. The income capitalization approach, if based on the assumption of the continuation of the current use into perpetuity, will not reconcile with the sales comparison approach. The appraiser can explain this condition since the sales comparison approach reflects the market's recognition that the current use does not reflect all the property's components of value. The analysis of the current use income indicates the current fundamental value, while the sales comparison approach reflects the sum of the current use (fundamental value) and the value of the options to use the property for other uses in the future. The cost approach will only reflect true overall value when the land comparables in the cost approach reflect the same real options concerning future highest and best uses. In rare cases, when law or contracts expand or limit options, the appraiser must explicitly adjust for option value in the calculations of functional and economic obsolescence or entrepreneurial profit for the improvements.

7. Dorothea M. Colwell and Peter F. Colwell, "The Timing of Development Revealed by the Market: An Options Approach," *The Appraisal Journal* (Spring 2004): 122. The Colwell and Colwell article shows financial options models can be used to find the term of the investment. In addition, the article shows the value relationship between a property with an option, and without an option and its overall value.

8. *The Dictionary of Real Estate Appraisal*, 5th ed., 104, defines *intrinsic value* as "the inherent worth of a thing; as contrasted with an empirical measure or an opinion of value, such as market value; a value considered to be inherently or internally associated with an object."

9. National Association of Certified Valuation Analysts and American Society of Appraisers, *International Glossary of Business Valuation Terms*, 2001, defines *intrinsic value* as "the value that an investor considers, on the basis of an evaluation or available facts, to be the 'true' or 'real' value that will become the market value when other investors reach the same conclusion. When the term applies to options, it is the difference between the exercise price or strike price of an option and the market value of the underlying security.

If the value of the fundamental real estate is equated to the value of the cash flow during the term of the current use, then the value of the option must be equal to the value of the reversion, which is a different use. This perspective allows appraisers to see that, while options exist in nearly all properties, redevelopment options are immaterial to the overall value in most cases, and the overall value is approximately equivalent, in most cases, to the value of the cash flow during the term of the current use. The net present value of the option is immaterially small because the reversion and change in use is far into the future.

Deterministic Modeling

There are several key concepts to be understood concerning real options, all of which will be familiar to appraisers. The first is that different decisions will create different values, yet the decisions are mutually exclusive. A developer cannot concurrently develop and hold for development at the same time. Secondly, circumstances and events happen and decisions are made at different points in time, and many decisions are made after the valuation date. Basic appraisal theory dictates that changing circumstances and their timings are best appraised using discounted cash flow (DCF) analysis with discounting and compounding. Real options valuation theory also extensively employs DCF modeling with discounting and compounding, but there is one major difference. Options valuation theory is said to explicitly address the uncertainty in decision making after the valuation date. Conversely, conventional appraisal using direct capitalization or traditional DCF analysis does not explicitly address the uncertainty surrounding each possible scenario and outcome, and it assumes that decision makers are passive after the appraisal date and throughout the analysis period. Conventional appraisal does not analyze the probability of, and the cash flow from, each option. Instead conventional appraisal analyzes only the typical, predominate, most likely, or most representative of the choices, and the risk associated with other possible outcomes is accounted for in the capitalization or discount rate used in the analysis. Conventional appraisal is therefore said to be deterministic while real options analysis is said to be probabilistic (stochastic).

Land Subdivision Feasibility

The following example illustrates the differences between deterministic modeling and real options modeling. Consider an analysis of a planned 12-home subdivision with the following facts: development will occur in two phases, each phase is two years, with 3 homes sold per year, first-year home prices are $400,000 and first-year total development costs are $375,000 per home with costs growing at 3% per year, and with yield expectations at 20%.

The first valuation model is a conventional appraisal model (Table 1). Since home value inflation is expected between 0% and 6% annually, the appraiser deterministically assumes values grow at 3% per year, and assumes, deterministically, that the developer completes both phases. This approach is deterministic for two reasons; the model does not address the potential for other future rates of growth for home prices, and it does not account for the possibility that the developer may choose to abandon the development before the start of phase two.

The deterministic model indicates that the development is unfeasible, because the net present value (NPV) is less than zero (negative $28,946) or because the internal rate of return (IRR) of 18% is less than the required yield rate of 20%. A real options analysis, however, will prove otherwise.

Real Options Model

The second model is a real options analysis known as a binomial model because the option is an "either/or," "yes/no" type of decision. Either the developer decides to develop phase two or decides not to. This type of model is sometimes called an *abandonment options model*. Figure 1 shows the decision tree for this appraisal problem.

There are also other types of real options models. Another common type is the trinomial model, which analyzes three option choices, abandon, exercise, or wait/postpone. If the Figure 1 example had three or more phases, the decision tree would be expanded to include a branch for each phase option. In reality, land developers often have the option of postponing phased developments, hence trinomial models are useful although more complex. Real estate disputes and litigation represent trinomial problems; litigate, do nothing, settle. Real estate management is trinomial: increase rents/lower occupancy, decrease rents/increase occupancy, or stay unchanged.

As shown in Table 2, the options binomial model assumes (1) home prices will either grow at 6% per year (Best Case Scenario) or there is no growth in prices (Worst Case Scenario), and (2) the developers

Table 1 Deterministic Model (Conventional Appraisal)

Year	1	2	3	4
Homes Built	6		6	
Homes Sold	3.00	3.00	3.00	3.00
Deterministic Inflation			3.0%	
Price per Home	$400,000	$412,000	$424,360	$437,091
Sales Revenue	$1,200,000	$1,236,000	$1,273,080	$1,311,272
Cost per Home	$375,000	$386,250	$397,838	$409,773
Total Costs	$2,250,000	$0	$2,387,025	$0
Cash Flow	–$1,050,000	$1,236,000	–$1,113,945	$1,311,272
Discount Rate	20%			
NPV	–$28,946			
IRR	**18%**			
Feasibility	**Unfeasible**			

Figure 1 Real Options Binomial Decision Tree

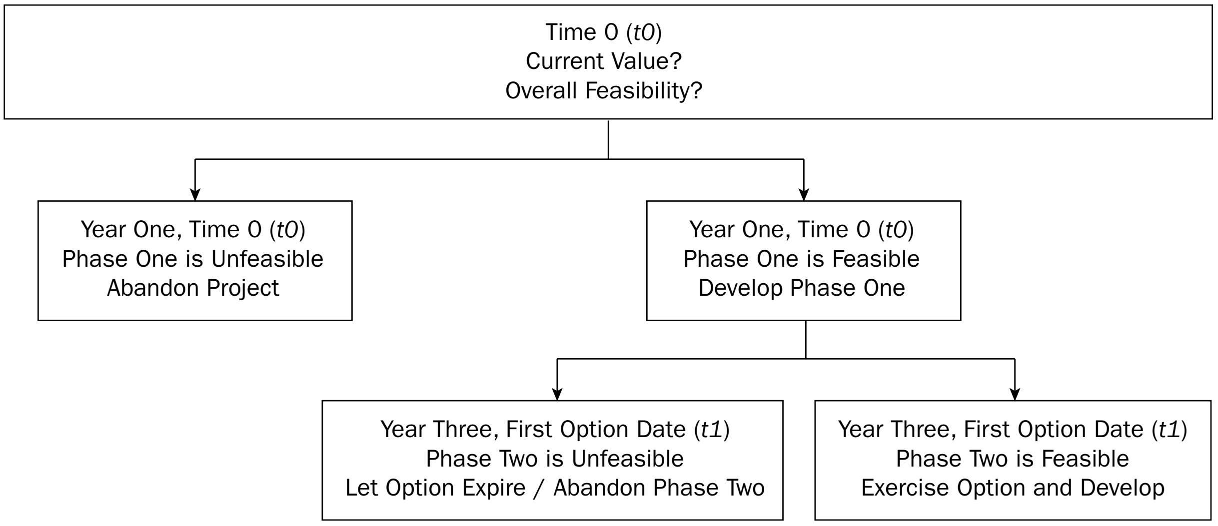

Table 2 Real Options Binomial Model

Best Case Scenario				
Year	1	2	3	4
Homes Built	6		6	
Homes Sold	3	3	3	3
Best Case Inflation			6%	
Price per Home	$400,000	$424,000	$449,440	$476,406
Sales Revenue	$1,200,000	$1,272,000	$1,348,320	$1,429,219
Cost per Home	$375,000	$386,250	$397,838	$409,773
Total Costs	$2,250,000	$0	$2,387,025	$0
Cash Flow	–$1,050,000	$1,272,000	–$1,038,705	$1,429,219
Discount Rate	20%			
NPV	$96,476			
IRR	27%			
Worst Case Scenario, Assuming Completion of Phase Two				
Year	1	2	3	4
Homes Built	6		6	
Homes Sold	3	3	3	3
Worst Case Inflation			0%	
Price per Home	$400,000	$400,000	$400,000	$400,000
Sales Revenue	$1,200,000	$1,200,000	$1,200,000	$1,200,000
Cost per Home	$375,000	$386,250	$397,838	$409,773
Total Costs	$2,250,000	$0	$2,387,025	$0
Cash Flow	–$1,050,000	$1,200,000	–$1,187,025	$1,200,000
Discount Rate	20%			
NPV	–$149,899			
IRR	8%			
Worst Case Scenario, Assuming Abandonment of Phase Two				
Year	1	2	3	4
Homes Built	6		–	
Homes Sold	3	3	–	–
Worst Case Inflation			0%	
Price per Home	$400,000	$400,000	$400,000	$400,000
Sales Revenue	$1,200,000	$1,200,000	$0	$0
Cost per Home	$375,000	$386,250	$397,838	$409,773
Total Costs	$2,250,000	$0	$0	$0
Cash Flow	–$1,050,000	$1,200,000	$0	$0
Discount Rate	20%			
NPV	–$41,667			
IRR	14%			

Table 3 Scenario Weighting

Best Case	50%			
Worst Case	50%			
Weighted Cash Flows, Assuming Completion of Phase Two				
Year	1	2	3	4
Weighted Cash Flow	–$1,050,000	$1,236,000	–$1,112,865	$1,314,610
Discount Rate	20%			
NPV	–$26,711			
IRR	**18%**			
Feasibility	**Unfeasible**			
Weighted Cash Flows, Assuming Abandonment of Phase Two				
Year	1	2	3	4
Weighted Cash Flow	–$1,050,000	$1,236,000	–$519,353	$714,610
Discount Rate	20%			
NPV	$27,405			
IRR	**23%**			
Feasibility	**Feasible**			

will make a decision whether to exercise the option to complete phase two development during the analysis period.

When the Best and Worst Case Scenarios are reviewed separately, results show that the Best Case Scenario is feasible with a positive NPV of $96,476.[10] The Worst Case Scenarios are not feasible because both have negative NPVs. If the developer exercises the option to complete phase two in the Worst Case Scenario, the development will yield an NPV of negative $149,899. Since, in the Worst Case Scenario, the development is closer to a positive NPV when the developer abandons phase two (this NPV is only negative $41,677), it is the prudent management option to abandon, rather than complete, phase two. The developer should let the option on phase two expire in this scenario.

To complete the binomial analysis, a solution from the Best Case and Worst Case-Abandonment Scenarios must be synthesized. Assuming a 50/50 chance for each scenario, the development is feasible when the option to develop phase two is abandoned in the Worst Case Scenario. (See Table 3 for the computations.)

For demonstration purposes, the weighting of the scenario of completing phase two when there is no home price inflation was computed and shown in Table 3. It demonstrates that the deterministic modeling of 3% inflation, yielding a NPV of negative $28,946 (Table 1), is approximately equivalent to the NPV (negative $26,711) of a probabilistic model of 50% chance of a 6% inflation rate and 50% chance of no inflation. This proves the ability of the option analysis to discover the value of the option to abandon phase two, which produces a weighted NPV of positive $27,405 (Table 3).

In this example, the conventional DCF incorrectly indicates that the development is unfeasible, but the real options analysis proves the development is feasible. If an appraiser had completed only a conventional DCF and had not completed a real options analysis, the appraiser would have drawn the wrong conclusions and would have incorrectly advised the client.

Monte Carlo Simulations

In many appraisal problems, the previous simplistic two-position assumption that inflation will be either

10. The scenario analyzing the DCF for the 6% growth but not developing phase two is not shown, because the developer will exercise the option to complete both phases in order to maximize profits given the adequate NPV and returns for each of the 6% scenarios.

6% or zero is adequate. Option theorists call this technique "fuzzy," because, while not a precisely accurate description of the possibilities, the fuzzy description of zero and 6% are good enough. For real estate appraisers, many appraisal problems can be adequately analyzed with fuzzy modeling.

In other cases, only a range of appraisal inputs for the scenarios will suffice. In practice, an appraiser could analyze any number of possible scenarios from within a range and could weight each scenario as indicated by market data. The development analyzed in the example could have been analyzed in terms of each whole percent between zero and six, or each half percent, each tenth, and so on. An analyst may wish to analyze thousands of price inflation scenarios. Furthermore, the weighting or probability of each scenario could be evenly distributed, as was done in Table 3, at 50/50. Alternatively, the appraiser could assume an uneven distribution of, for example, 25% chance of zero inflation and say 75% chance of an inflation rate of 6%. Or, the appraiser could assume a probability distribution of scenarios from an area under a normal (bell) curve,[11] or from some other type of curve or slope.

Unlike the simple problem in the example, where only the probability of home price inflation is tested, real-life appraisal problems often require the testing of multiple key inputs. Even in the simplistic model, it would not be unreasonable to also want to analyze a range of starting home prices and costs, absorption rates, inflation rates, and yield expectations. Obviously this math work can quickly become impractical for many appraisal assignments. However, some assignments require the analysis of thousands of scenarios.

Monte Carlo simulation is an analysis where multiple inputs and scenarios are analyzed, from dozens to tens of thousands of times. Real estate appraisers sometimes intuitively begin a Monte Carlo simulation analysis when, for example, they test their DCFs by computing different values for different discount rates, income or expense growth rates, or changes in occupancy, etc. Then deterministically, they report only the predominate or typical one. To properly complete a Monte Carlo simulation, the appraiser would need to complete hundreds or thousands of scenarios, and compile and analyze the results.

Specialized software, such as Crystal Ball published by Oracle Corporation, is often used for Monte Carlo simulation work. This software allows the appraiser to analyze the real-life possible ranges of numerous appraisal inputs, forecasts, and expectations simultaneously. Once the Monte Carlo simulation is completed, the software provides the appraiser with value, yield, and other performance statistics, which are reconciled with other approaches and analyses to make value conclusions. A Monte Carlo analysis is one step in a real options analysis, albeit a sometimes monstrous computational step.

Discount Rates in Real Options Valuation

Many analysts use lowered discount rates in their DCFs when they conduct real options scenarios, compared to the rates they would use in a deterministic model. The idea is that in a deterministic model there is greater risk and uncertainty that the actual cash flow will not materialize as forecasted, because the deterministic model is likely to be wrong since it is an average, typical, or predominate forecast. It is argued that each of the scenarios in real option models should be discounted at its safe rate,[12] because the risk of the failure of each of the forecasts to materialize (either by substantially outperforming or underperforming the forecast)—and the impact on the value of a failure to materialize—is accounted for in the selection and weighting process for each forecast, and therefore, it does not need to be accounted for in the discount rate.

Closed-Form Models

Black-Scholes-Merton Options Pricing Model

The land development example presented previously employed a generalized approach to real options valuation. The well-known Black-Scholes (also called Black-Scholes-Merton or BSM[13]) options pricing model is a special case formula that applies to real options concerning financial options. The formula is simple to use and is closed-form in that

11. *The Appraisal of Real Estate*, 13th ed., 605.

12. *The Dictionary of Real Estate Appraisal*, 5th ed., 175, defines *safe rate* as "the minimum rate of return on invested capital. Theoretically, the difference between the total rate of return and the safe rate is considered a premium to compensate the investor for risk, the burden of management, and the illiquidity of the capital invested; also called *riskless rate* or *relatively riskless rate*."

13. Fischer Black and Myron Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* 81, no. 3 (May-June 1973): 637—654. Robert C. Merton was another major contributor to the closed-form model. There are many other important contributors both before and after the popularization of BSM.

the inputs and computations are finite in nature and well-defined in valuation theory. The BSM model was a major development in the valuation of financial options, although today numerous extensions, enhancements, and variations have been discovered and rediscovered. A closed-form model can be employed to value real estate purchase options.

Tables 4 and 5 present a closed-form options pricing model with classic financial markets notation, along with translations to conventional real estate appraisal notation. Real estate appraisers will need to translate their terminology and market measurements into terms and measurements that can be inserted into the standard closed-form models.

Note that exponentials and natural logarithms are types of compounding and discounting formulas and are how financial option analysts compute these formulas.[14] Using conventional appraisal compounding and discounting formulas will produce different, often incorrect results, depending on whether the various rates are all periodic or continuous.

The BSM model assumes a normal bell curve for the chances of the scenarios. In real estate, both skewness[15] and kurtosis[16] occur on many occasions. While the basic BSM model has difficulty with scenarios in which the potential outcomes form anything other than a normal distribution, special adjustments to the basic BSM model can be made for skewness or kurtosis. These adjustments will not be discussed here. In any case, the appraiser can address these issues in the reconciliation, as would be done with any other methodology.

Current Market Value, Exercise Price, and Term

Current market value (V_o) is equivalent to the stock price (S_0) in the closed-form model. It is estimated by the real estate appraiser via traditional methods; it is the value today under the assumption of the use and rights that property has at the option expiration date. The exercise price (K) and term (T) are provided in the option contract. Alternatively, the appraiser could estimate the exercise price (K) as the current market value (V_o) grown at the appropriate inflation factor for the term (T). For land subdivision potential, the current market value (V_o or S_o), could be perceived as the value of the land, assuming approvals are not obtained, and the exercise price (K)—the market value (V_o) at the exercise date—is the value of the

Table 4 Closed-Form Model Terminology

Closed-Form Model		Comparison	Conventional Appraisal	
Current (*t0*) Stock Price	S_0	Equivalent to	Current Market Value of Property	V_o
Exercise or Strike Price, at a future date (*t1*)	K	Equivalent to	Exercise, Strike or Contract Price of Property, at a future date (*t1*)	K
Volatility (probabilistic)	σ	Related to	Expected Change in Market Value over Term (*t1-t0*) (deterministic)	$\Delta\%$
Riskless Rate	r	Related to	Yield Rate	Y_o
		Equivalent to	Safe, Riskless or Risk-Free Yield Rate	Y_s
Term	T	Equivalent to	Term	T
Annual Dividend Yield	q	Equivalent to	Dividend Yield Rate on Cash Flows, but not Reversion	Y_{cf}
Standard Normal Cumulative Distribution	Φ	Equivalent to	Standard Normal Cumulative Distribution	Φ

14. Exponential and natural logarithm formulas differ from conventional appraisal compounding and discounting in that conventional appraisal formulas are periodic while exponentials and natural logarithms are continuous. Conventional appraisal formulas typically are limited to periods of annually, semi-annually, quarterly, and monthly. Continuous compounding and discounting assumes that the number of periods approaches infinity.

15. A skewed curve means one tail of the bell curve is longer than the other.

16. Kurtosis describes the relationship between the height and width of the peak of a bell curve and the height of the tails of the curve.

Table 5 Closed-Form Model, Intermediate and Final Steps

Intermediate Steps	
Step 1: Present Value of Exercise Price	$= K_0$
	$= K \times$ Exponential of $(-r * T)$
Step 2: S_0 Discounted for Annual Dividends	$= S_{0\text{-}q}$
	$= S_0 \times$ Exponential of $(-q * T)$
Step 3: $d1$	$=$ (Natural Logarithm$(S_0/K) + (r - q + \sigma\hat{}2/2) \times T)/(\sigma \times$ Square Root(T))
Step 4: $d2$	$= d1 - (\sigma \times$ Square Root$(T))$
Final Steps	
Step 4: Value of Call Option	$= c$
	$= S_{0\text{-}q} \times$ Standard Normal Cumulative Distribution$(d1)$ $- K_0 \times$ Standard Normal Cumulative Distribution$(d2)$
Step 5: Value of Put Option	$= p$
	$= K_0 \times$ Standard Normal Cumulative Distribution$(-d2)$ $- S_{0\text{-}q} \times$ Standard Normal Cumulative Distribution$(-d1)$

Note: *d1* and *d2* are the traditional names within the options literature for the derived intermediate steps in the BSM calculations.

land with the use and rights that the property will have after the approvals. If these assumptions about value are made, then corresponding assumptions about risk, volatility, and dividends must be made.

Volatility

Volatility (sigma, σ) in the closed-form model describes the potential range of the change in the future price of a stock. It is the measure of how much values could change. It is analogous to an appraiser's time adjustment (delta, Δ%). The difference between sigma and delta is that sigma is probabilistic and delta is typically deterministic. Sigma describes a range of possible outcomes, and delta describes the typical, average, or predominate outcome. Most often, volatility is derived by analyzing historical changes in the stock markets, which have large quantities of data to analyze. Stock analysts use numerous statistical methods to estimate volatility, often including standard deviation, variances, and confidence interval statistics. Stock volatility can be extracted from daily to yearly comparisons and then quoted on an annualized basis. Real estate appraisers rarely have statistically significant quantities of data. Still, real estate appraisers can find good foundations for probabilistic value change forecasts on which to compute a closed-form model.

There are two kinds of volatility measured in the market: implied and historical. Implied volatility is solved using an option pricing model, such as a BSM model, and current stock and option prices. Historical volatility is measured statistically, commonly as the standard deviation in historical stock prices.

Real Estate Volatility

Table 6 shows the classically calculated volatility of several real estate indices. They are based on the standard deviation for the time periods shown.

While helpful, complex statistics are not required to employ the closed-form model. However, the real estate appraiser will need to unpack the deterministic assumptions from their conventional time adjustments. For example, if an appraiser uses a deterministic 4% time adjustment in the sales comparison or income capitalization approach, the appraiser will need to reexamine the source data for the time adjustment to estimate the range of possible time adjustments. Assume the appraiser finds that the meaningful range of potential change in value is from –5% to 10% annually, and the mean is 4%. Then, the indicated volatility is approximately 7.5% (the average of 9% (104% – 95%) and 6% (110% – 104%)). Numerous other methods can be used. Additionally, just as the stock analysts employ standard deviation, variance, and confidence interval methods to control extreme data, real estate appraisers are entitled to limit their ranges to reasonably probative and practical ranges and are not required to blindly reflect all possible ranges of price changes indicated

Table 6 Real Estate Volatility

	Case-Shiller Home Price Indices		FTSE NAREIT US Index Series
	CSXR*	SPCS20R†	All REITs
Last 12 months	4.5%	4.7%	16.0%
Last 36 months	3.6%	3.7%	18.2%
Last 10 years	4.3%	4.2%	24.2%
Since beginning	(1987) 3.2%	(2000) 3.9%	(1971) 18.0%

* S&P/Case-Shiller 10-City Composite Home Price Index

† S&P/Case-Shiller 20-City Composite Home Price Index

by the limited raw data. Appraisal judgment must be employed, and the appraiser must be aware that this input is the most difficult to understand and estimate.

Riskless, Risk-Free Rates

Riskless rates are a type of yield rate. Since the closed-form model accounts for a major portion of the total risk in the volatility component, the input for general risk should exclude the risk captured in the volatility component. The remaining risk is described as riskless or risk-free, which is a misnomer, especially for real estate. It would be more accurate and descriptive to label this as the low-risk benchmark or the low-risk alternative. It is the yield rate from the low-risk category of alternative investments and is often the rate for government bonds or AAA corporate bonds. The risk refers to the risk that the yield will not be as expected due to risks other than volatility. The riskless rate for real estate is usually higher than what would be used for stock and options valuation, due to the illiquidity, maturity risk premium, lack of diversification, management intensity, and other risks associated with real estate investment.

Real Estate Purchase Option Example

The role of the components of the closed-form model can be illustrated in the following real estate purchase option example.

In this example, assume a current market value of $1,000,000, an exercise price of $1,050,000, a term of six months (0.5), a riskless rate of 5.0% and a volatility estimate of 7.5%. Using this information, an appraiser can estimate the value of an option as shown in Table 7. Note that dividend yields are

Table 7 Real Estate Purchase Option

Inputs		
Stock Price, current (*t0*)	S_0	$1,000,000
Exercise or Strike Price, future (*t1*)	K	$1,050,000
Volatility	σ	7.5%
Riskless Rate	r	5.0%
Term	T	0.5
Annual Dividend Yield	q	0.0%
Intermediate Steps		
Present value of K, at r, $T = K_0$	K_0	$1,024,075
$d1$ = (Natural Logarithm(S_0/K) + ($r - q + \sigma$^2/2) × T)/(σ × Square Root(T))	$d1$	(0.422075)
$d2 = d1 - $ (σ × Square Root(T))	$d2$	(0.475108)
Final Step		
Call Option Value = S_0 × Standard Normal Cumulative Distribution($d1$) – K_0 × Standard Normal Cumulative Distribution($d2$)	c	$11,490

usually not expected in real estate purchase options and therefore are included in the model at 0.0%.

Type of Purchase Options

The BSM solves European options, which allow the exercise of the option only at the end of the option period. Many real estate purchase options may be better valued using American option models, such as a binomial model, which allow the exercise at any time during the term.

Conclusion

Real options analysis and financial option pricing models can and should be applied to real estate valuation, because they improve the accounting for value and risk. The theory and practice is well established in financial markets and business valuation. Real estate valuation opportunities include real estate purchase options, development options, abandonment and feasibility studies, and many other major real estate decisions. Special knowledge is required as the analyses require important adaptations and special uses of conventional real estate data.

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Additional Reading

American Society of Appraisers. *Valuing Machinery and Equipment: The Fundamentals of Appraising Machinery and Technical Assets*, 2nd ed. Washington DC: American Society of Appraisers, 2005.

Kummer, Donald R., and Arthur L. Schwartz, Jr. "Valuing Real Property Purchase Options." *The Real Estate Appraiser and Analyst* 46, no. 1 (January-February 1980): 13–17.

Web Connections

Internet resources suggested by the Y. T. and Louise Lee Lum Library

ABA Real Property, Probate and Trust Journal—A Primer on Real Estate Options
http://online.dakotahomestead.com/Vol%20XI/Articles-%5BO%5D-Options-APrimeronREOptions.pdf

Chicago Board Options Exchange (CBOE)—Options Basics Tutorial
http://www.cboe.com/LearnCenter/Tutorials.aspx#basics

Designing Property Futures Contracts and Options Based on NCREIF Property Indices
(Available to Appraisal Institute individuals by contacting the Lum Library)
http://law-journals-books.vlex.com/vid/designing-futures-options-ncreif-indices-62972804

Mortgage Professional American: Real Estate Option Contract Explained for Investors
http://www.mpamag.com/real-estate/real-estate-option-contract-explained-for-investors-14835.aspx

Realtor.com—How to Use a Real Estate Option Contract
http://www.realtor.com/home-finance/homebuyer-information/how-to-use-a-real-estate-option-contract.aspx?source=web